Math 312

SURFACE INTEGRAL QUESTIONS.

1. Evaluate $\iint_S yz dS$ where S is the plane surface given by the equations $x \geq 0$, $y \geq 0$, $z \geq 0$, $x + y + z = 1$.

2. Evaluate $\iint_S z^2 dS$ where S is the part of the cone $x^2 + y^2 = 2z^2$ which lies between the planes $z = 1$ and $z = 3$.

3. Let S_1 represent the sphere $x^2 + y^2 + z^2 = a^2$ and S_2 represent the hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$. Evaluate the following integrals.

   (a) $\iint_{S_1} z^2 dS$ (b) $\iint_{S_2} z dS$ (c) $\iint_{S_1} y^2 z^2 dS$ (d) $\iint_{S_1} (z+3)\, dS$

4. If $S : x^2 + y^2 = 4, 0 \leq z \leq 3$, evaluate

   (a) $\iint_S (1 + x^2)\, dS$ (b) $\iint_S (x + z)\, dS$ (c) $\iint_S y^2 z^3 dS$

5. Find the surface area of

   (a) $z^2 = 3(x^2 + y^2), 0 \leq z \leq b$
   (b) the cylinder $x^2 + y^2 = 1$ between $z = 0$ and $z = 1 + y$
   (c) the triangle $x + 2y + 3z = 6$, $x, y, z \geq 0$
   (d) the part of the hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$ inside $x^2 + y^2 = b^2$ $(0 < b < a)$
   (e) the ellipse formed by cutting the plane $2x + 3y + 6z = 60$ with the cylinder $x^2 + y^2 = 2x$.

6. Evaluate $\iint_S (x^2 + y^2 + z^2)\, dS$ where $S : x^2 + y^2 + z^2 = a^2$. Deduce $\iint_S x^2 dS$